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$\therefore$ An anharmonic ratio $\dfrac{AM}{MN}:\dfrac{AB}{BN}=\dfrac{c^2-y_1^2}{x_1^2-c^2}$; but $c^2=\dfrac{a^2b^2}{a^2+b^2}$.

$\therefore$ An anharmonic ratio$=\dfrac{b^2}{a^2}\cdot\dfrac{(a^2+b^2)x_1^2-a^2b^2}{(a^2+b^2)x_1^2-a^2b^2}=\dfrac{b^2}{a^2}$.

Also solved by G. B. M. Zerr.

CALCULUS.

181. Proposed by **S. F. NORRIS, Baltimore, Md.**

Integrate $dy=\dfrac{x^2dx}{1+x^4}$. [From Olney's *Integral Calculus*, page 116, third example, second part].

Solution by **G. W. GREENWOOD, M. A.; M. E. GRABER, M. A., and G. B. M. ZERR, A. M., Ph. D.**

$$\frac{x^2}{1+x^4}=\frac{1}{2\sqrt{2}}\left[\frac{x}{x^2-x\sqrt{2}+1}-\frac{x}{x^2+x\sqrt{2}+1}\right]$$

$$=\frac{1}{4\sqrt{2}}\left[\frac{(2x-\sqrt{2})+\sqrt{2}}{x^2-x\sqrt{2}+1}-\frac{(2x+\sqrt{2})-\sqrt{2}}{x^2+x\sqrt{2}+1}\right]$$

$$=\frac{1}{4\sqrt{2}}\left[\frac{2x-\sqrt{2}}{x^2-x\sqrt{2}+1}-\frac{2x+\sqrt{2}}{x^2+x\sqrt{2}+1}\right]$$

$$+\tfrac{1}{4}\left[\frac{1}{x^2-x\sqrt{2}+1}+\frac{1}{x^2+x\sqrt{2}+1}\right].$$

The required integral is therefore

$$=\frac{1}{4\sqrt{2}}[\log(x^2-x\sqrt{2}+1)-\log(x^2+x\sqrt{2}+1)]$$

$$+\frac{\sqrt{2}}{4}[\tan^{-1}(\sqrt{2}x-1)+\tan^{-1}(\sqrt{2}x+1]$$

$$=\frac{1}{4\sqrt{2}}\log\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}+\frac{\sqrt{2}}{4}\tan^{-1}\frac{x\sqrt{2}}{1-x^2}.$$

Also solved by J. Scheffer.

182. Proposed by **A. H. HOLMES, Brunswick, Maine.**

Evaluate $\displaystyle\int_0^{\frac{1}{2}\pi}d\theta\sqrt{[1+\sin^2\theta(1-4\cos\theta)]}$.

Solution by S. A. COREY, Hiteman, Iowa.

$$\int_0^{\frac{1}{2}\pi} d\theta\sqrt{[1+\sin^2\theta(1-4\cos\theta)]} \equiv \int_0^{\frac{1}{2}\pi} d\theta\sqrt{(1\tfrac{1}{2}-\cos\theta-\tfrac{1}{2}\cos 2\theta+\cos 3\theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

The value of the second member of (1) is easily computed by aid of the formula

$$f(x)-f(0)=\frac{x}{m.2}\left\{[f'(x)+f'(0)]+2\left[f\left(\frac{x}{m}\right)+f'\left(\frac{2x}{m}\right)+f'\left(\frac{3x}{m}\right)+\ldots\ldots\ldots\right.\right.$$

$$\left.\left.\ldots\ldots\ldots+f'\left(\frac{m-1}{m}x\right)\right]\right\}-\frac{B_2x^2}{m^2.2!}[f'(x)-f'(0)]+\frac{B_2x^4}{m^4.4!}[f^{\text{iv}}(x)-f^{\text{iv}}(0)]+\ldots\ldots\ldots$$

$$\ldots\ldots\ldots+(-1)^n\frac{B^nx^{2n}}{m^{2n}.(2n)!}[f^{2n}(x)-f^{2n}(0)]+\ldots\ldots\ldots$$

B_1, B_2, ..., being Bernoulli's numbers.

Taking $m=5$, and $n=2$, the value of the definite integral is found to be 1.17066. For greater accuracy, larger values of m and n may be taken.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $\sin^2\theta(1-4\cos\theta)=x$. Then by expanding the expression we get

$$(1+x)^{\frac{1}{2}}=1+\tfrac{1}{2}x-\tfrac{1}{8}x^2+\tfrac{1}{16}x^3-\tfrac{5}{128}x^4+\tfrac{7}{256}x^5-\ldots\ldots\ldots$$

$$\int_0^{\frac{1}{2}\pi} d\theta=\tfrac{1}{2}\pi=1.5708; \qquad \tfrac{1}{2}\int_0^{\frac{1}{2}\pi} xd\theta=\tfrac{1}{8}\pi-\tfrac{2}{3}=-.2740;$$

$$-\tfrac{1}{8}\int_0^{\frac{1}{2}\pi} x^2d\theta=-\frac{11\pi}{128}=-.0699; \qquad \tfrac{1}{16}\int_0^{\frac{1}{2}\pi} x^3d\theta=\frac{35\pi}{512}-\tfrac{59}{252}=-.0194;$$

$$-\tfrac{5}{128}\int_0^{\frac{1}{2}\pi} x^4d\theta=\tfrac{215}{792}-\frac{2975\pi}{32768}=-.0138;$$

$$\tfrac{7}{256}\int_0^{\frac{1}{2}\pi} x^5d\theta=\frac{16401\pi}{131072}-\frac{54761}{137280}=-.0059;$$

$$\therefore \int_0^{\frac{1}{2}\pi}\sqrt{[1+\sin^2\theta(1-4\cos\theta)]}d\theta=1.1858-.$$

Also solved by the Proposer.

MECHANICS.

170. Proposed by ELISHA S. LOOMIS, Ph. D., Berea, Ohio.

Two angles of iron, A_1CD and A_1CA_3, move freely on a pivot at C. Rods B_1A_1 and B_1A_3 are attached respectively at A_1 and at some point A_3 so that